Bearing-Only Super-Twisting Control for Multi-Quadrotor Formations

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***Abstract*—This paper addresses the bearing-only formation tracking control problem for a group of Quadrotors. We propose a novel control strategy applicable to double-integrator agents, enabling them to maintain a desired formation while tracking leaders with time-varying trajectories, which are unknown for followers. The approach relies on bearing-only measurements and considers disturbances affecting the agents motion. A key contribution is the design and stability analysis of a sliding mode control law based on the Super-Twisting Algorithm (STA), which enhances robustness against disturbances. A performance comparison is presented between a baseline bearing-only controller and the STA-based sliding mode controller, evaluated using formation accuracy and disturbance rejection metrics. Results demonstrate the superior robustness, adaptability, and tracking precision of the proposed method. The effectiveness of the proposed approach is shown through experiments in the realistic simulator Gazebo.**

***Index Terms*—Bearing-only formation control, Quadrotor systems, Sliding mode control, Multi-agent tracking.**

I. Introduction

In recent decades, cooperative control in multi-agent systems has garnered significant interest due to its applications in civilian and military domains [1]. Notable applications include geometric pattern generation in controlled environments [2], target localization and tracking [3], and mapping and exploration tasks [4].

The fundamental objective of cooperative control is to design control laws that integrate individual agent dynamics with interaction topologies to achieve collective objectives. Multi-robot systems offer key advantages, such as fault tolerance, scalability, and cost efficiency in complex tasks. The bearing rigidity theory for formations and graphs has been developed to determine whether a geometric pattern can be achieved through directional constraints, known as bearings [5].

The bearing-based approach uses relative position measurements to achieve the desired formation, defined by angular constraints [6], leveraging a key mathematical concept: the orthogonal projection matrix. This method relies exclusively on the direction of neighboring agents, without requiring exact position or distance data, and can be applied to agents modeled as single or double integrators, or unicycles [7]. Additionally, it addresses practical challenges like formation tracking, providing flexibility in uncertain and disturbed environments. This approach facilitates navigation algorithms and tracking techniques with reduced reliance on precise measurements, making it particularly useful in dynamic environments, such as urban areas or scenarios with variable speeds.

Supervising leader agents with unknown velocities adds complexity, as followers must adjust their trajectories without precise information about the leaders' future movements. In [8], a control strategy based solely on relative orientation is proposed, eliminating the need for absolute distances and assuming constant-velocity leaders sharing a local reference frame. In [9], leader velocities vary over time, and the inclusion of a signum function in their control law introduces discontinuities, leading to chattering-like behavior. While high gain values are used to ensure convergence, in practice, this may cause actuator failures due to oscillations, vibrations, and mechanical stress from excessive current draw. Other techniques, such as sliding mode control, have been applied to multi-agent tracking problems, including integral sliding mode [10] and classical sliding mode control, which guarantees sliding surface convergence in bearing-based approaches [11]. An analysis of chattering in [12] demonstrates that the Super-Twisting Algorithm (STA) significantly reduces this effect while preserving the robustness of sliding mode techniques.

However, in uncontrolled environments, a critical challenge is the lack of access to global positioning systems (e.g., GPS), which limits knowledge of the agents' absolute positions. To address this problem, control strategies have been developed that minimize reliance on sensing, allowing operation with partial information, especially in scenarios involving unknown

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target tracking.

In this paper, a control scheme is proposed for trajectory tracking of a formation of Quad-Rotors, adopting a predefined geometric pattern. The methodology is based on Sliding Mode Control techniques, using bearing-only measurements, which enables robust trajectory tracking by controlling the desired velocities for each Quad-Rotor. The effectiveness of the proposed approach is validated through simulations conducted in the Gazebo environment.

This document is organized as follows: Section II presents the preliminaries and the problem statement; in Section III, the robust control strategy and its stability analysis are described. The experimental validation of this proposal in Gazebo is detailed in Section IV. Finally, Section V presents the conclusions.

II. PRELIMINARIES

This section presents some background on graph theory, the basics on rigidity theory, and defines the problem addressed.

***Definition* 1.** *[7]. An undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ consists of nodes $\mathcal{V}$ (agents) and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. For edge $(i,j) \in \mathcal{E}$, agents measure relative bearings mutually. The neighborhood $\mathcal{N}_i = \{k \in \mathcal{V} : (i,k) \in \mathcal{E}\}$.*

A formation $(\mathcal{G}, \boldsymbol{p})$ maps vertices to positions $\boldsymbol{p} \in \mathbb{R}^{n3\times 1}$. For edge $k = (i,j)$:

$$\boldsymbol{e}_k = \boldsymbol{e}_{ij} = \boldsymbol{p}_j - \boldsymbol{p}_i, \quad \boldsymbol{g}_k = \boldsymbol{g}_{ij} = \frac{\boldsymbol{e}_k}{\|\boldsymbol{e}_k\|} \tag{1}$$

with stacked vectors $\boldsymbol{e} = [\boldsymbol{e}_1^T, ..., \boldsymbol{e}_k^T]^T$, $\boldsymbol{g} = [\boldsymbol{g}_1^T, ..., \boldsymbol{g}_k^T]^T$.

***Definition* 2.** *The bearing function $F_B : \mathbb{R}^{n3} \to \mathbb{R}^{m3}$ for formation $(\mathcal{G}, \boldsymbol{p})$ is:*

$$F_B(\boldsymbol{p}) = [\boldsymbol{g}_1^T, ..., \boldsymbol{g}_k^T]^T \tag{2}$$

encoding relative bearing constraints between agents.

***Definition* 3.** *[13]. The bearing rigidity matrix $R_B(\boldsymbol{p}) \in \mathbb{R}^{m\times n3}$ is:*

$$R_B(\boldsymbol{p}) = \frac{\partial F_B(\boldsymbol{p})}{\partial \boldsymbol{p}} \tag{3}$$

with bearing rates $\dot{\boldsymbol{g}}_k = \frac{\mathbf{P}_{g_k}}{\|\boldsymbol{e}_k\|}\dot{\boldsymbol{e}}_k$, where $\mathbf{P}_{g_k} = \mathbf{I}_d - \boldsymbol{g}_k \boldsymbol{g}_k^T$ is the orthogonal projection matrix. All the above concepts are represented in Fig. 1.

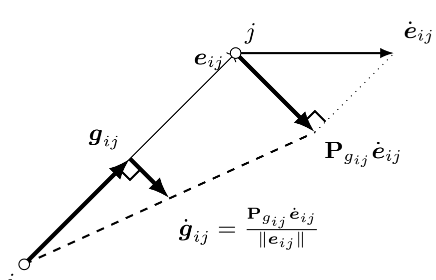

Fig. 1. Geometric relationship between $\boldsymbol{g}_{ij}$ and $\boldsymbol{e}_{ij}$. Source [7]

***Definition* 4.** *The incidence matrix $\mathbf{H} \in \mathbb{R}^{m\times n}$ of an oriented graph satisfies:*

$$\boldsymbol{e} = (\mathbf{H} \otimes \mathbf{I}_d)\boldsymbol{p}, \quad \dot{\boldsymbol{e}} = \bar{\mathbf{H}}\dot{\boldsymbol{p}} \tag{4}$$

where $\otimes$ is the Kronecker product and $\bar{\mathbf{H}} \in \mathbb{R}^{m3\times n3}$. For connected graphs, rank$(\mathbf{H}) = n-1$ [14], where $[\mathbf{H}]_{ki} = -1$ if node i is the tail of edge j, $[\mathbf{H}]_{kj} = 1$ if node i is the head of edge j, other elements are 0.

Consider a network $(\mathcal{G}, \boldsymbol{p})$ with no located nodes, the bearing Laplacian matrix $\mathcal{B} \in \mathbb{R}^{n3\times n3}$ is defined as [13]:

$$[\mathcal{B}]_{ij} = \begin{cases} 0_{3\times 3} & i \neq j, (i,j) \notin \mathcal{E}, \\ -\mathbf{P}_{g_{ij}} & i \neq j, (i,j) \in \mathcal{E}, \\ \sum_{k\in\mathcal{N}_i} \mathbf{P}_{g_{ik}} & i = j, i \in \mathcal{V}. \end{cases} \tag{5}$$

In a network with n_l leaders and $n_f = n - n_l$ followers the bearing Laplacian matrix can also be separated into the following sub-matrices

$$\mathcal{B} = \begin{bmatrix} \mathcal{B}_{ll} & \mathcal{B}_{fl} \\ \mathcal{B}_{lf} & \mathcal{B}_{ff} \end{bmatrix} \tag{6}$$

where $\mathcal{B}_{ff} \in \mathbb{R}^{n_f 3\times n_f 3}$ shows the interactions between followers.

A. Model

Consider a system composed of n Quad-Rotors in $\mathbb{R}^3$, where n_l are leaders and $n_f = n - n_l$ are followers. The leaders have the following mathematical model:

$$\dot{\boldsymbol{p}}_i = \boldsymbol{v}_r, \quad i \in \{1, ..., n_l\} \tag{7}$$

where $\boldsymbol{p}_i \in \mathbb{R}^3$ is the i-th position of the leader agent in the global frame and $\boldsymbol{v}_r \in \mathbb{R}^3$ is the velocity reference.

The remaining n_f Quad-Rotors have a double integrator dynamic, described by the following equation:

$$\begin{aligned} \dot{\boldsymbol{p}}_i &= \boldsymbol{v}_i \\ \dot{\boldsymbol{v}}_i &= \boldsymbol{u}_i + \boldsymbol{\Delta}_i \end{aligned} \tag{8}$$

with $i = 1,\ 2,\ ...,\ n = \overline{1,n}$ where, Here, $\boldsymbol{p}_i = [x_i\ y_i\ z_i]^T \in \mathbb{R}^3$ represents the position, with $x_i, y_i \in \mathbb{R}$ as horizontal coordinates and $z_i \in \mathbb{R}$ as vertical position. $\boldsymbol{v}_i = [\dot{x}_i\ \dot{y}_i\ \dot{z}_i]^T \in \mathbb{R}^3$ is the velocity and $\boldsymbol{u}_i$ the control input to be designed. The term $\boldsymbol{\Delta}_i = [d_{x_i}\ d_{y_i}\ d_{z_i}]^T \in \mathbb{R}^3$ denotes disturbances, such as unmodeled dynamics and wind gusts.

B. Problem Statement

The n agents consist of n_l leaders, which move with a time-varying velocity $\boldsymbol{v}_r(t)$, and n_f followers. The objective for the followers is to achieve a desired position $\boldsymbol{p}^*(t)$ such that $\boldsymbol{p}(t) \to \boldsymbol{p}^*(t)$ as $t \to \infty$. This desired position depends on the current bearings $\boldsymbol{g}_{ij}(t)$ with respect to desired bearings $\boldsymbol{g}_{ij}^*$.

Let $(\mathcal{G}, \boldsymbol{p}^*)$ be the desired target formation in $\mathbb{R}^3$ which satisfies the following constraints:

1) Bearings: $\boldsymbol{g}_{ij} = \boldsymbol{g}_{ij}^*, \quad \forall (i,j) \in \mathcal{E}$
2) Velocity: $\boldsymbol{v}_i(t) = \boldsymbol{v}_r(t)$

Leader's velocity is unknown for followers.

The following assumptions are introduced to facilitate the resolution of the problem statement.

Assumption 1. *The disturbances are uniformly bounded and Lipschitz continuous. That is,* $d_{x_i} \in \mathcal{L}_{D_{x_i}}$, $d_{y_i} \in \mathcal{L}_{D_{y_i}}$, $d_{z_i} \in \mathcal{L}_{D_{z_i}}$, $\dot{d}_{x_i} \in \mathcal{L}_{\bar{D}_{x_i}}$, $\dot{d}_{y_i} \in \mathcal{L}_{\bar{D}_{y_i}}$, $\dot{d}_{z_i} \in \mathcal{L}_{\bar{D}_{z_i}}$, $d_{\phi_i} \in \mathcal{L}_{D_{\phi_i}}$, *with positive constants* D_{x_i}, D_{y_i}, D_{z_i}, $\bar{D}_{x_i}$, $\bar{D}_{y_i}$, $\bar{D}_{z_i}$.

Assumption 2. *The target formation* $(\mathcal{G}, \boldsymbol{p}^*)$ *is infinitesimally bearing rigid in* $\mathbb{R}^3$, *and the number of leaders satisfies* $n_l \geq 2$ *[15], where* n_l *agents are leaders and the rest* $n_f = n - n_l$ *agents are the followers.*

Assumption 3. *Each agent can measure the bearing vector* $\boldsymbol{g}_k(t)$ *of its neighbors, but no absolute position nor velocity measurements are available.*

The following lemma is introduced in the case of leader-follower formation.

Lemma 1 (Unique target formation [5], [7])**.** *The target configuration* $(\mathcal{G}, \boldsymbol{p}^*)$ *can be uniquely determined by the bearings* $\boldsymbol{g}_k^*$ *and the states of the fixed leaders positions* $\boldsymbol{p}_i^*$ *if and only if the matrix* $\mathcal{B}_{ff}$ *appearing in (6) is non-singular.*

III. Robust bearing-only formation control

In this section, the design of the proposed motion coordination control strategy for a team of quad-rotors is addressed, where a robust strategy based on continuous sliding-mode control (STA) is presented.

The proposed formation control design considers a group of n Quad-Rotors. The dynamics of each Quad-Rotor, which are inherently nonlinear and supposed to be perturbed can be approximated by a double integrator model [16], [17]. This simplification might be obtained through linearization around a hovering equilibrium including a gravity compensation, where the position and yaw angle behave as a set of independent double integrators, described by:

$$\begin{aligned} \dot{\boldsymbol{p}}(t) &= \boldsymbol{v}(t), \\ \dot{\boldsymbol{v}}(t) &= \boldsymbol{u}(t) + \boldsymbol{\Delta}(t), \end{aligned} \tag{9}$$

where $\boldsymbol{p} \in \mathbb{R}^{n3}$ is the stacked position vector of all agents, $\dot{\boldsymbol{v}} \in \mathbb{R}^{n3}$ represents the stacked velocity vector, $\boldsymbol{u} \in \mathbb{R}^{n3}$ is the control input to be designed, and $\boldsymbol{\Delta}(t) \in \mathbb{R}^{n3}$ denotes an unknown bounded perturbation.

The objective is to design a robust control input $\boldsymbol{u}$ that ensures finite-time convergence of the sliding variable to zero, stabilizes the system trajectories, and achieves the desired formation under the influence of perturbations.

A. Control Design

Let $\boldsymbol{g} \in \mathbb{R}^{m3}$ and $\boldsymbol{g}^* \in \mathbb{R}^{m3}$ be the stacked vectors of actual and desired bearings, respectively, and $\bar{\mathbf{H}}^T$ the incidence matrix. The formation error dynamics are defined as:

$$\begin{aligned} \mathrm{e} &= \bar{\mathbf{H}}^T(\boldsymbol{g} - \boldsymbol{g}^*), \\ \boldsymbol{\varepsilon} &= \dot{\mathrm{e}} = \bar{\mathbf{H}}^T\dot{\boldsymbol{g}} = \bar{\mathbf{H}}^T\mathrm{diag}(\mathbf{P}_{\boldsymbol{g}})\bar{\mathbf{H}}\dot{\boldsymbol{p}}. \end{aligned} \tag{10}$$

Differentiating Equation (10) yields:

$$\begin{aligned} \dot{\mathrm{e}} &= \boldsymbol{\varepsilon}, \\ \dot{\boldsymbol{\varepsilon}} &= \bar{\mathbf{H}}^T\mathrm{diag}(\mathbf{P}_{\boldsymbol{g}})\bar{\mathbf{H}}\dot{\boldsymbol{v}}. \end{aligned} \tag{11}$$

Substituting $\dot{\boldsymbol{v}}$ from Equation (9) leads to:

$$\begin{aligned} \dot{\mathrm{e}} &= \boldsymbol{\varepsilon}, \\ \dot{\boldsymbol{\varepsilon}} &= \mathbf{M}(\boldsymbol{u} + \boldsymbol{\Delta}), \end{aligned} \tag{12}$$

where $\mathbf{M} = \bar{\mathbf{H}}^T\mathrm{diag}(\mathbf{P}_{\boldsymbol{g}})\bar{\mathbf{H}}$. The aim is to design the control $\boldsymbol{u}$ that guarantees convergence and rejects disturbances. To achieve zero in the dynamics the control input in the form of a super-twisting controller is considered to obtain the desired behavior, this is $\mathrm{e} = 0$, then $\dot{\mathrm{e}} = \boldsymbol{\varepsilon} = 0$. $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{s})$ is selected with the super-twisting algorithm.

To provide a comparative analysis of the proposed controller's performance against a commonly used approach in the literature, we first briefly introduce the Proportional-Derivative (PD) Controller.

B. Baseline Proportional-Derivative Controller (PD) [5]

A bearing-only Proportional-Derivative Controller (PD) is defined as:

$$\boldsymbol{u}_j = k_{j0}\mathrm{e}_j + k_{j1}\boldsymbol{\varepsilon}_j \tag{13}$$

where $k_{j0},\ k_{j1} > 0$ are control gains and $j = x,\ y,\ z$.

C. Bearing-only Super-Twisting Algorithm (STA)

The Super-Twisting Algorithm (STA) is defined as:

$$\begin{aligned} \boldsymbol{s}_j &= \boldsymbol{\varepsilon}_j + k_{j0}\mathrm{e}_j, \\ \boldsymbol{u}_j &= \boldsymbol{\nu}_j - k_{j1}\lceil \boldsymbol{s}_j \rfloor^{\frac{1}{2}}, \\ \dot{\boldsymbol{\nu}} &= -k_{j2}\lceil \boldsymbol{s}_j \rfloor^{0}, \end{aligned} \tag{14}$$

where $\lceil \boldsymbol{s} \rfloor^p = |\boldsymbol{s}|^p\,\mathrm{sign}\,(\boldsymbol{s})$, $k_{j0},\ k_{j1},\ k_{j2} > 0$ are control gains and $j = x,\ y,\ z$, and a guide for selection of gains is presented in [18].

Assumption 4. *The control laws presented in* (14) *act on the high-level controller of each quad-rotor. It is assumed that the low-level controller of each agent ensures convergence to zero.*

Assumption 5. *During formation evolution, no collisions occur between neighboring agents.*

Remark 1. *The control inputs* $\boldsymbol{u}_k$ *are robust against bounded and Lipschitz continuous disturbances. Additionally, these signals are continuous and the chattering problem is attenuated.*

The following theorem is presented to establish the convergence of the system (12) under the assumptions that the desired formation is infinitesimally bearing rigid.

Theorem 1. *Consider the multi-agent system (9), the formation errors* e, $\boldsymbol{\varepsilon}$ *defined in (10) and the error dynamics (12). The control law (14) guarantees that* $\mathrm{e} \to 0$ *and* $\boldsymbol{\varepsilon} \to 0$ *as* $t \to \infty$, *which yields that* $\boldsymbol{g} \to \boldsymbol{g}^*$ *and consequently* $\boldsymbol{p} \to \boldsymbol{p}^*$ *and* $\boldsymbol{v} \to \boldsymbol{v}^*$.

Proof: We first prove the finite-time convergence of the error dynamics to the sliding surface $\boldsymbol{s} = 0$. Consider the time derivative of $\boldsymbol{s}_k$ (given in (14); for this general case, $\boldsymbol{s}_k = \boldsymbol{s}$). Substituting (11), we obtain:

$$\begin{aligned}\dot{\boldsymbol{s}} &= \dot{\boldsymbol{\varepsilon}} + k_0\dot{\mathbf{e}},\\ &= \mathbf{M}(\boldsymbol{u}+\boldsymbol{\Delta}) + k_0\mathbf{M}\dot{\boldsymbol{p}},\\ &= \mathbf{M}\boldsymbol{u} + \boldsymbol{\Delta}_1 + k_0\mathbf{M}\dot{\boldsymbol{p}},\end{aligned} \tag{15}$$

where $\mathbf{M} = \bar{\mathbf{H}}^T\mathrm{diag}(\mathbf{P}_{\boldsymbol{g}})\bar{\mathbf{H}} \in \mathbb{R}^{n3\times n3}$ and $\boldsymbol{\Delta}_1 = \mathbf{M}\boldsymbol{\Delta}$. Consider the Lyapunov function:

$$V = \frac{1}{2}\boldsymbol{s}^T\boldsymbol{s}. \tag{16}$$

Taking its time derivative and substituting the previous expressions yields $\dot{V} = \boldsymbol{s}^T\dot{\boldsymbol{s}} < 0$, and $\boldsymbol{s} \neq 0$, then

$$\begin{aligned}\dot{V} &= \boldsymbol{s}^T(k_0\mathbf{M}\dot{\boldsymbol{p}} + \mathbf{M}\boldsymbol{u} + \boldsymbol{\Delta}_1)\\ &= \boldsymbol{s}^T\mathbf{M}\boldsymbol{u} + \boldsymbol{s}^T\boldsymbol{\Delta}_2\end{aligned} \tag{17}$$

where $\boldsymbol{\Delta}_2 = k_0\mathbf{M}\dot{\boldsymbol{p}} + \boldsymbol{\Delta}_1$. The term $\mathbf{M}\dot{\boldsymbol{p}}$ can be treated as a disturbance under Assumption 3. Substituting the control input $\boldsymbol{u}(\boldsymbol{s})$ from (14), we get:

$$\boldsymbol{s}^T\dot{\boldsymbol{s}} = \boldsymbol{s}^T\mathbf{M}\left(-k_1\lceil\boldsymbol{s}\rfloor^{\frac{1}{2}} - \int_0^t k_2\lceil\boldsymbol{s}\rfloor^0\, d\tau\right) + \boldsymbol{s}^T\boldsymbol{\Delta}_2. \tag{18}$$

Under Assumption 1, $\boldsymbol{\Delta}_2$ is bounded, i.e., $|\boldsymbol{\Delta}_2| \leq D$. To ensure $\dot{V} \leq 0$, we substitute (18) into $\dot{V}$:

$$\begin{aligned}\dot{V} &= \boldsymbol{s}^T\mathbf{M}\left(-k_1\lceil\boldsymbol{s}\rfloor^{\frac{1}{2}} - \int_0^t k_2\lceil\boldsymbol{s}\rfloor^0\, d\tau\right) + \boldsymbol{s}^T\boldsymbol{\Delta}_2,\\ &= -k_1|\boldsymbol{s}^T||\mathbf{M}||\boldsymbol{s}|^{1/2} - |\boldsymbol{s}^T|\left(|\mathbf{M}|\int_0^t k_2\, d\tau - D\right),\\ &\leq -k_1|\boldsymbol{s}^T||\mathbf{M}||\boldsymbol{s}|^{1/2},\\ &\leq 0.\end{aligned} \tag{19}$$

Since $\dot{V} < 0$ for $\boldsymbol{s} \neq 0$, is negative definite and global asymptotic stability of the system can be guaranteed.

Now, let us prove that the convergence to the sliding surface is reached in finite-time and given that $\boldsymbol{s} = \boldsymbol{\varepsilon} + k_{l0}\mathbf{e}$. From the candidate Lyapunov function $V = \boldsymbol{s}^T\boldsymbol{s} = \|\boldsymbol{s}\| = \sqrt{2V}$, we have

$$\begin{aligned}\dot{V} &\leq -k_1|\boldsymbol{s}^T||\mathbf{M}||\boldsymbol{s}|^{1/2}\\ &\leq -k_1|\mathbf{M}||\boldsymbol{s}|^{3/2}\\ &\leq -k_1|\mathbf{M}|(2V)^{3/4}\\ \dot{V} &\leq -\alpha V^{\theta}\end{aligned} \tag{20}$$

where $\alpha = k_1|\mathbf{M}|2^{3/4}$ and $\theta = 3/4$. If the Lyapunov function satisfies the above differential equation for some $\alpha > 0$ and $0 < \theta < 1$, the system converges to the sliding surface in a settling time T expressed as follows

$$T(\boldsymbol{s}_0) \leq \frac{V(\boldsymbol{s}_0)^{1-\theta}}{\alpha(1-\theta)}. \tag{21}$$

Once the convergence to the sliding surface $\boldsymbol{s} = 0$ is guaranteed and given the definition of the surface, we have $k_0\mathbf{e} + \boldsymbol{\varepsilon} = 0$. As $\boldsymbol{\varepsilon} = \dot{\mathbf{e}}$, therefore

$$\dot{\mathbf{e}} = -k_0\mathbf{e}. \tag{22}$$

The above expression represents a line in the phase plane $(\mathbf{e}, \dot{\mathbf{e}})$ that crosses zero and its slope depends on the constant k_0. The solution of this equation is:

$$\mathbf{e}(t) = \mathbf{e}(0)\mathbf{e}^{-k_0 t}. \tag{23}$$

This means that once the error dynamics (12) has reached the sliding surface $\boldsymbol{s} = 0$, both formation errors $\mathbf{e}, \boldsymbol{\varepsilon}$ converge to zero exponentially with $k_0 > 0$ and the initial condition $(\mathbf{e}(0))$ of the agents can be arbitrary in the three-dimensional space.

As the proposed sliding variable is defined in terms of bearing errors, convergence of the sliding variable $\boldsymbol{s} \to 0$ implies that $\boldsymbol{g} \to \boldsymbol{g}^*$; given the assumptions that the desired bearing formation $(\mathcal{G}, \boldsymbol{p}^*)$ is infinitesimally bearing rigid and there is no collisions between agents, as a direct consequence $\boldsymbol{v} \to \boldsymbol{v}^*$ and $\boldsymbol{p} \to \boldsymbol{p}^*$ and there exist sufficient leaders to ensure a unique target formation according to Assumption 2 and Lemma 1. ■

IV. Experimental results

To validate the proposed controller, we tested its performance on a simulated swarm of ten Parrot Bebop 2 Quad-Rotors, the first two of them are considered leaders, and the remaining eight are followers. The simulation environment was developed using ROS Rolling and Gazebo Ionic on Ubuntu 24.10. This setup allowed us to create a realistic and controllable environment for evaluating the controller's effectiveness. The initial conditions of the agents are:

$$\begin{aligned}
&\mathbf{R}_1 = [\ 1.0,\ 1.0,\ 5.0]^\top, && \mathbf{R}_6 = [-0.1,\ 1.2,\ 5.0]^\top,\\
&\mathbf{R}_2 = [-1.0,\ 1.0,\ 5.0]^\top, && \mathbf{R}_7 = [-1.52, -0.7,\ 5.0]^\top,\\
&\mathbf{R}_3 = [-2.52, -0.7,\ 5.0]^\top, && \mathbf{R}_8 = [\ 0.5, -1.88,\ 5.0]^\top,\\
&\mathbf{R}_4 = [\ 2.1, -1.88,\ 5.0]^\top, && \mathbf{R}_9 = [\ 1.72, -0.7,\ 5.0]^\top,\\
&\mathbf{R}_5 = [\ 2.72, -0.7,\ 5.0]^\top, && \mathbf{R}_{10} = [\ 1.8,\ 1.2,\ 5.0]^\top.
\end{aligned}$$

The leader Quad-Rotors velocity are given by: For all t:

$$\boldsymbol{v}_1 = \boldsymbol{v}_2 = \begin{bmatrix} 0.03 + 0.075\sin(0.05t)\\ 0.075\cos(0.05t) + 0.075\sin(2t)\\ 0.048\end{bmatrix}.$$

Except:
$20 < t \leq 24$:

$$\boldsymbol{v}_1 = \begin{bmatrix} 0.018 + 0.012\cos\left(\pi\frac{t-20}{2}\right) + 0.075\sin(0.05t)\\ 0.075\cos(0.05t) + 0.075\sin(2t)\\ 0.048\end{bmatrix}$$

$50 < t \leq 54$:

$$\boldsymbol{v}_1 = \begin{bmatrix} 0.06 - 0.03\cos\left(\pi\frac{t-50}{2}\right) + 0.075\sin(0.05t)\\ 0.075\cos(0.05t) + 0.075\sin(2t)\\ 0.048\end{bmatrix}.$$

The desired formation, shown in Figure 2, is defined by the relative positions of each robot, denoted as $\mathbf{R}_i^* \in \mathbb{R}^3$. These base positions are:

$$\begin{aligned}
&\mathbf{R}_1^* = [\ 1.0,\ 1.0,\ 3.0]^\top, && \mathbf{R}_6^* = [-1.0,\ 1.0,\ 1.0]^\top,\\
&\mathbf{R}_2^* = [-1.0,\ 1.0,\ 3.0]^\top, && \mathbf{R}_7^* = [-1.62, -0.9,\ 1.0]^\top,\\
&\mathbf{R}_3^* = [-1.62, -0.9,\ 3.0]^\top, && \mathbf{R}_8^* = [\ 0.0, -2.08,\ 1.0]^\top,\\
&\mathbf{R}_4^* = [\ 0.0, -2.08,\ 3.0]^\top, && \mathbf{R}_9^* = [\ 1.62, -0.9,\ 1.0]^\top,\\
&\mathbf{R}_5^* = [\ 1.62, -0.9,\ 3.0]^\top, && \mathbf{R}_{10}^* = [\ 1.0,\ 1.0,\ 1.0]^\top.
\end{aligned}$$

The desired bearing vector from robot i to robot j, denoted as $\boldsymbol{g}_{ij}^*$, is calculated from these desired positions:

$$\boldsymbol{g}_{ij,d} = \frac{\boldsymbol{p}_{j,\text{desired}} - \boldsymbol{p}_{i,\text{desired}}}{\|\boldsymbol{p}_{j,\text{desired}} - \boldsymbol{p}_{i,\text{desired}}\|}. \tag{24}$$

This calculation is performed for all distinct pairs of robots (i, j) where $i \neq j$. The interaction topology and communication links among the robots within the formation, shown in Figure 2, are defined by the adjacency matrix A.

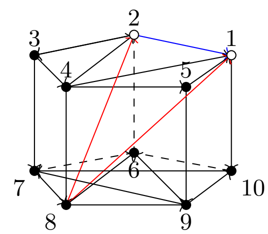

Fig. 2. Communication topology.

For an undirected graph, $\mathbf{A}_{ij} = \mathbf{A}_{ji} = 1$ if robots i and j are connected, and 0 otherwise, and this is the formation we expect to maintain with the group of agents.

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

The selected gains for each controller are detailed in Table I. Figures 4 and 5 show the trajectory tracking behavior of the ten agents, and Figure 6 shows the behavior of the control signals of the agents.

TABLE I
STA AND PD CONTROLLER GAINS

Controller	Axis	k_0	k_1	k_2
STA (14)	x, y	0.15	0.9	0.0015
	z	0.8	1.0	0.0012
PD (13)	x, y	5.12	8.7	-
	z	4.12	7.7	-

The bearing error of the agents is evaluated by the metric defined in (25), and its evolution over time is depicted in Figure 3

$$\boldsymbol{e}_m = \sqrt{\frac{\sum_{(i,j)\in\mathcal{V}}(e_{x,i,j}^2 + e_{y,i,j}^2 + e_{x,i,j}^2))}{3n_f}}. \tag{25}$$

To evaluate the performance of the proposed controller and demonstrate its effectiveness, we compared it against an existing Proportional-Derivative (PD) controller presented in [7], a scheme validated for bearing-only formation tracking of second order agents.

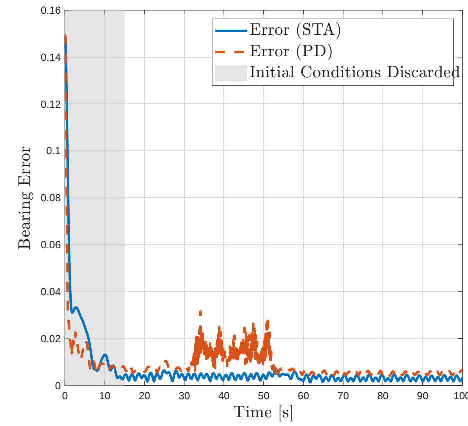

Fig. 3. Bearing Error Comparative.

The average bearing error (in RMS) of the agents was calculated for each controller. Additionally, the minimum, mean, and maximum RMS performance indices were considered for each controller. The best results are highlighted in bold in Table II. Data from an initial time $t_0 = 15$ [s] and a final time $t_f = 100$ [s] were considered to prevent the indices from being affected by the initial conditions of the Quad-Rotors.

An experiment similar to the one presented in this section can be seen in the following video: https://youtu.be/pix3_PgBlzo

TABLE II
GLOBAL RMS PERFORMANCE COMPARISON

Controller	Minimum	Median	Maximum
STA Global RMS	**0.0014**	**0.0036**	**0.0070**
PD Global RMS	0.0040	0.0081	0.0316

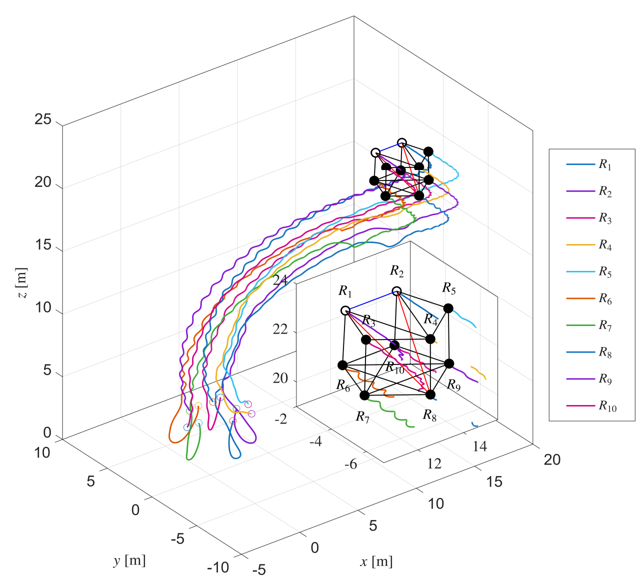

Fig. 4. Trajectories in 3D with a zoom in the final part.

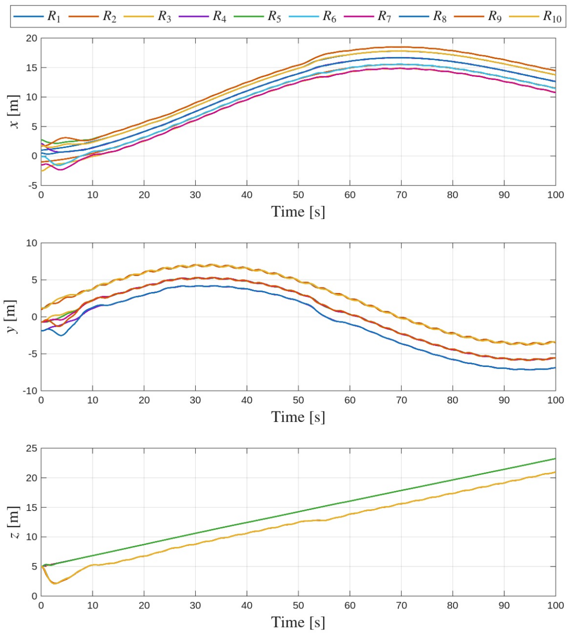

Fig. 5. Trajectories with respect to time.

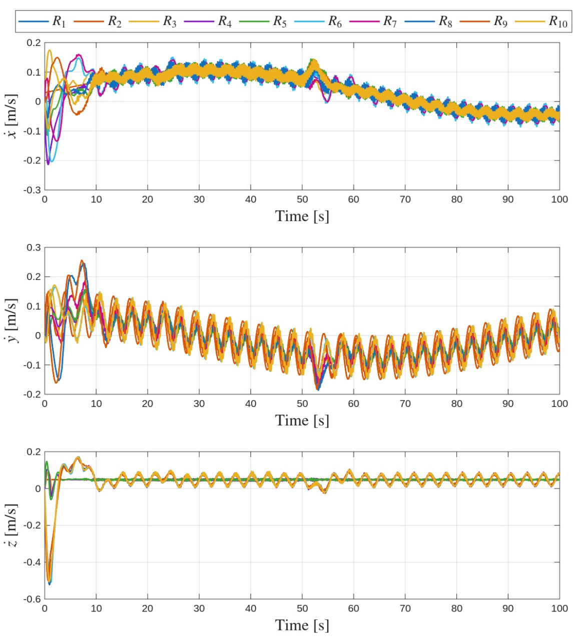

Fig. 6. Control inputs with respect to time.

V. Conclusions

In this work, we designed and validated a robust control strategy for a group of Quad-Rotors using a Continuous Sliding Mode control law (Super-Twisting Algorithm). To achieve this, two leaders with variable speeds and eight follower agents were employed, allowing for the effective evaluation of this control strategy in the formation and tracking of the Quad-Rotors. It was observed that the proposed Super-Twisting Algorithm (STA) controller showed better performance compared to the conventional control strategy (PD), which demonstrated the effectiveness of the proposed controller. As future work, we plan to implement the proposed approach using real Quad-Rotors, relying on estimation of bearings from visual information provided by cameras onboard the agents.

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